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CS 326

4/23/2022

HW 6

1. TreeVC(G):

C = empty set;

While G has edges:

Pick vertex v w/ degree 1

Let w be the other end of edge touching v

Join w to C

Remove all edges touching w from G

Return C

This should work because any nonempty tree has at least one degree 1 vertex v and any cover must contain v or w, but w touches all edges touched by v so it is a good replacement.

1. Assume no three points are in a line together. To cross itself, the tour must have two edges crossing in the interior. We can get a shorter tour by uncrossing the edges, and this shows that a shortest tour has no crossing edges. This is thanks to the triangle inequality property.
2. a) The verifier for this should be expecting a certificate that is a subcollection C. This verifier should check that C is a subset of F, that every element of X is in at least one of the sets in C, and that the number of sets = k. Once all of those are true, then the verifier can say that the set cover is in NP, and all of this can be done in polytime.

b) Given a vertex cover instance (G, k) with expected output of set cover instance (X, F, k’) where F contains a cover of X of size k’ and G has a vertex cover of size k, we can do a polytime reduction as follows:

Reduction(G=(V,E), k):

k’ = k

X = E

F = {{e in E such that e touches v} for every v in V}

Return (X, F, k’)

1. Claim: If y is in Pi and y <= t, then there is a z in Li with y/(1+ε/2n)^I <= z <= y

Base (i=0): P0 = L0 = <0>, so choose z = y = 0

Inductive step (0 < i <= n):

Pick y in Pi such that y <= t. Now y=y’ or y=y’ + Xi for y’ in Pi-1. Since y’ <= t, there is a z’ in Li-1 so that y’/(1+ ε/2n)^i-1 <= z’ <= y’. Let z” = z’ + (y-y’), then z” is in MERGELISTS(Li-1, Li-1 + xo) = L, and L is the input to TRIM(L, delta), delta = ε/2n. This means that the output L’ of TRIM(L, delta) contains some z such that z <= z” <= (1+delta)z. All of this leads to the following:

z = z” = z’ + (y-y’) <= y’ + (y-y’) = y

z >= z”/(1+delta) = (z’ + (y-y’))/(1+delta)

= z’/(1+delta) + (y-y’)/(1+delta)

>= y’/(1+delta)^i + (y-y’)/(1+delta)

>= y/(1+delta)^i = y/(1+ ε/2n)^i.

z will ultimately go on to Li since z <= y <= t.